Genetic Algorithms

Dr. Mahmoud Nabil Mahmoud
mnmahmoud@ncat.edu

North Carolina A & T State University

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Outline

Multi-Objective Optimization Problems

- Involve more than one objective function that are to be minimized or maximized
- Answer is set of solutions that define the best tradeoff between competing objectives

Ex.

- Suppose a company would like to design a phone that achieve the following objectives:
  - Short charging time $f_1(\mathbf{x})$
  - Small phone size $f_2(\mathbf{x})$
- $\mathbf{x} =$ (type of material, no of cells, $\ldots$)

General Form of MOOP

- Mathematically:

$$\begin{aligned} \texttt{min/max} \quad & f_m(\mathbf{x}) && m = 1, \ldots, M \\ \texttt{subject to} \quad & g_j(\mathbf{x}) >= 0 && j = 1, \ldots, J \\ \texttt{where} \quad & L_i <= x_i <= U_i && i = 1, \ldots, n \end{aligned}$$

- $\mathbf{x} = (x_1, \ldots, x_n)^T$
- $F(\mathbf{x}) = (f_1(\mathbf{x}), \ldots, f_M(\mathbf{x}))^T$

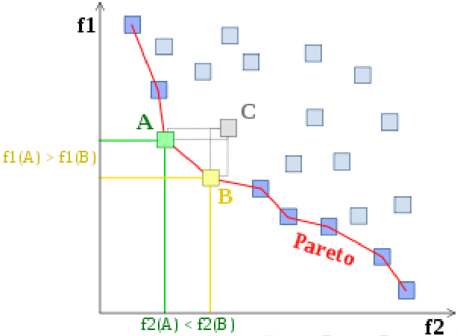
Define: Feasible Solution space, Feasible objective space, Utopia Point

Pareto optimal solutions

- In multi-objective optimization, there does not typically exist a feasible solution that minimizes all objective functions simultaneously.
- Therefore, attention is paid to Pareto optimal solutions that is, solutions that cannot be improved in any of the objectives without degrading at least one of the other objectives.

Ex. minimize f_1 and minimize f_2

- A dominates C
- B dominates C
- Points A and B are not strictly dominated by any other.

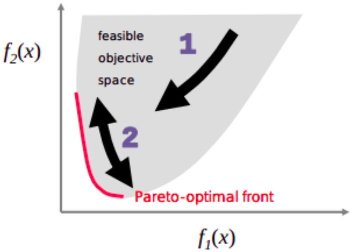


Dominance

- A solution x is called Pareto optimal, if there does not exist another solution that dominates it. The set of Pareto optimal outcomes is often called the Pareto front, Pareto frontier, or Pareto boundary.
- In multi-objective optimization problem, the goodness of a solution is determined by the dominance
- **Dominance Test**
  - x_1 dominates x_2, if
    - Solution x_1 is better than or as good as solution x_2 in all objectives
  - x_1 dominates x_2 or x_2 is dominated by x_1

Goals in MOO

- Find set of solutions as close as possible to Pareto optimal front
- To find a set of solutions as diverse as possible



Outline

Weighted Sum Method

- Scalarize a set of objectives into a single objective by adding each objective pre-multiplied by a user supplied weight

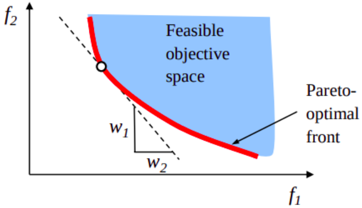
$$\begin{aligned} \texttt{minimize} \quad & F(x) = \sum_{m=1}^{M} w_m f_m(\mathbf{x}) \\ \texttt{subject to} \quad & g_j(\mathbf{x}) >= 0 \qquad j = 1, \ldots, J \\ \texttt{where} \quad & L_i <= x_i <= U_i \qquad i = 1, \ldots, n \end{aligned}$$

- Weight of an objective is chosen in proportion to the relative importance of the objective

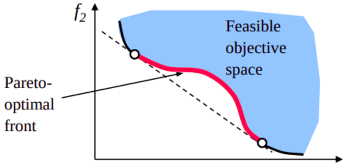
Weighted Sum Method

- **Advantage**
  - Simple
- **Disadvantage**
  - It is difficult to set the weight vectors to obtain a Pareto-optimal solution in a desired region in the objective space.
  - It cannot find certain Pareto-optimal solutions in the case of a non-convex objective space

Weighted Sum Method (Convex Case)



Weighted Sum Method (Non-Convex Case)



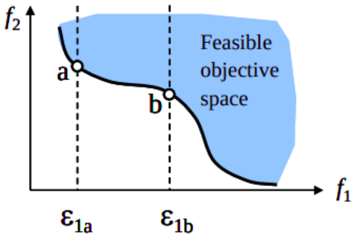
ϵ-Constraint Method

- Keep just one of the objective and restricting the rest of the objectives within user-specific values

$$
\begin{aligned}
\texttt{minimize} \quad & f_a(x), & \\
\texttt{subject to} \quad & f_m(x) <= \epsilon_m & m = 1, \ldots, M \text{ and } m \neq a \\
& g_j(\mathbf{x}) >= 0 & j = 1, \ldots, J \\
\texttt{where} \quad & L_i <= x_i <= U_i & i = 1, \ldots, n
\end{aligned}
$$

ϵ-Constraint Method

- Keep f_2 as an objective **Minimize** $f_2(x)$
- Treat f_1 as a constraint $\epsilon_a \leq f_1(x) \leq \epsilon_b$



ϵ-Constraint Method

- Advantage
  - Applicable to either convex or non-convex problems
- Disadvantage
  - The ϵ vector has to be chosen carefully so that it is within the minimum or maximum values of the individual objective function

Outline

1. Introduction
2. Classical Approaches
3. Genetic Algorithms

Introduction

- There are several different multi-objective evolutionary algorithms.
- The idea is to evolve a set of individual solution to approach the Pareto frontier
- One of the famous GA for multi-objective optimization problems is Non dominated Sorting Genetic Algorithm (NSGA-II).

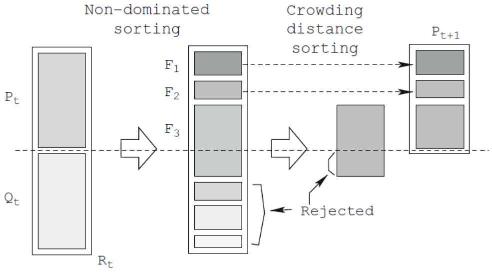
Non dominated Sorting Genetic Algorithm (NSGA-II)

NSGA-II is one evolutionary algorithm that has the following three features:

- It uses an elitist principle , i.e. the elites of a population are given the opportunity to be carried to the next generation.
- Is uses an explicit diversity preserving mechanism (Crowding distance)
- It emphasizes the non-dominated solutions.

How it works?

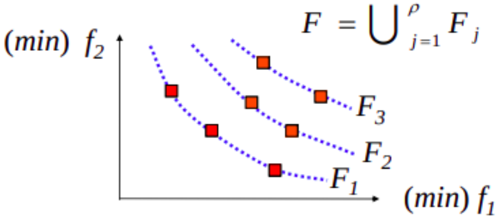
1. **Step 1** $R_t = P_t \cup Q_t$ Perform non dominated sorting to R_t and identify different frontiers F_i
2. **Step 2** $P_{t+1} = \phi$ i =1
   - Repeat until $|P_{t+1}| + |F_i| < N \Rightarrow |P_{t+1}| = |P_{t+1}| \cup F_i \quad i = i + 1$
3. **Step 3** Perform crowd sorting on last F_i and include most widely spread solutions $(N - P_{t+1})$
4. **Step 4** Q_{t+1} from P_{t+1} using crowding distance tournament selection



How it works?

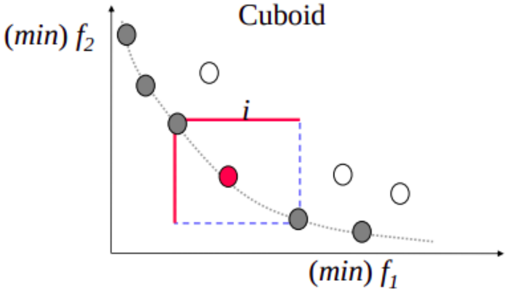
Non-Dominated Sorting

- Non-Dominated Sorting
  - Classify the solutions into a number of mutually exclusive equivalent non-dominated sets



Crowding Distance

Determine the density of solutions surrounding a particular solution



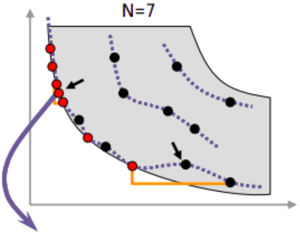
known as crowding distance.

Crowding Tournment Selection

- Assume that every solution has a non-domination rank and a local crowding distance
- A solution i wins a tournament with another solution j
  1. If the solution i has a better rank
  2. They have the same rank but solution i has a better crowing distance than solution j

NSGA II

- **Advantages**
  - The diversity among non-dominated solutions is maintained using the crowding procedure: No extra diversity control is needed
  - Elitism protects an already found Pareto-optimal solution from being deleted
- **Disadvantages**
  - When there are more than N members in the first non-dominated set, some Pareto-optimal solutions may give their places to other non-Pareto-optimal solutions



A Pareto-optimal solution is discarded

References

- Goldenberg, D.E., 1989. Genetic algorithms in search, optimization and machine learning.
- Michalewicz, Z., 2013. Genetic algorithms + data structures= evolution programs. Springer Science & Business Media

Questions